LABORATORIUM KIMIA FISIKA
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MATEMATIKA KIMIA
Persamaan Differensial

(Sumber : Barrante, Applied Mathematics fo Physical Chemistry, Bab 6)

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1. Solve the following linear differential equations:

   (a) $\frac{dy}{dx} + 3y = 0$

   (b) $\frac{dy}{dx} - 3y = 0$

   (c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

   (d) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

   (e) $\frac{d^2y}{dx^2} + 9y = 0$

   (f) $\frac{dx}{dt} = k_1(a - x) - k_2x$; k_1, k_2, and a are constants.

   (g) $\frac{d\phi}{dr} = -a\phi$; a is constant.

   (h) $\frac{d(A)}{(A)} = -k\,dt$; k is constant.

   (i) $\frac{1}{\Phi(\phi)}\frac{d^2\Phi}{d\phi^2} = -m^2$; m is constant.

   (j) $m\frac{d^2y}{dt^2} = -ky$; m and k are constants.

   (k) $\frac{d^2\psi}{dx^2} + \frac{8\pi^2mE}{h^2}\psi = 0$; E, m, and h are constants.

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2. Test the following differentials for exactness:
   (a) $dF = 2xy^2\,dx + 2yx^2\,dy$
   (b) $dF = 8x\,dx$
   (c) $dF = 12x^2y\,dx + 4x^3\,dy$
   (d) $dF = 5\,dx$
   (e) $dF = \frac{1}{y}\,dx - \frac{x}{y^2}\,dy$
   (f) $dF = xy\,dx + x^3\,dy$
   (g) $dP = \frac{nR}{V}\,dT - \frac{nRT}{V^2}\,dV$; n and R are constants.
   (h) $dV = \pi r^2\,dh + 2\pi rh\,dr$
   (i) $dq = nC_v\,dT + \frac{nRT}{V}\,dV$; n, C_v, and R are constants.
   (j) $d\rho = -\frac{PM}{RT^2}\,dT + \frac{M}{RT}\,dP$; M and R are constants.
   (k) $dE = nC_v\,dT + \frac{n^2a}{V^2}\,dV$; n, C_v, and a are constants

3. Show that the differential $dq = nC_v\,dT + (nRT/V)\,dV$, where n, C_v, and R are constants, can be made exact by multiplying by an integrating factor $1/T$. The resulting differential dS is called the *differential entropy change*.

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3. Show that the differential $dq = nC_v\,dT + (nRT/V)\,dV$, where n, C_v, and R are constants, can be made exact by multiplying by an integrating factor $1/T$. The resulting differential dS is called the *differential entropy change*.
4. Show that if $\sin 3x$ and $\cos 3x$ are particular solutions to the differential equation

$$\frac{d^2y}{dx^2} + 9y = 0$$

then a linear combination of the two solutions also is a solution.
5. Bessel's equation is an important differential equation having the general form

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - c^2)y = 0$$

where c is a constant. Find the indicial equation for the series solution to this equation.
6. A one-dimensional harmonic oscillator is described classically by the equation

$$\frac{d^2y}{dt^2} + 4\pi^2\nu^2 y = 0$$

Show that a solution to this equation is $y = A \sin 2\pi\nu t$, where A, π, and ν are constants.
7. The differential equation describing the spacial behavior of a one-dimensional wave is

$$\frac{d^2f}{dx^2} + \frac{4\pi^2}{\lambda^2}f(x) = 0$$

where λ is the wavelength. Find the general solution to this equation.
8. *Boundary conditions* are special restrictions imposed on the solutions to differential equations. The boundary conditions for a plucked string bound at both ends between $x = 0$ and $x = L$, and described by the equation given in Problem 7, are that $f(x)$ goes to 0 at $x = 0$ and $x = L$. Show how these boundary conditions affect the solution to the equation in Problem 7.

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9. Show that if we let $x = \cos\theta$, the solution to the associated Legendre's equation [Equation (6-47)] is $y = \cos\theta$, when $\ell = 1$ and $m = 0$, and is $y = \sin\theta$, when $\ell = 1$ and $m = 1$.

10. Show that if we let $x = \cos\theta$, the solution to the associated Legendre's equation [Equation (6-47)] is $y = \frac{1}{2}(3\cos^2\theta - 1)$, when $\ell = 2$ and $m = 0$.

11. The Schrödinger equation for a particle in a three-dimensional box is

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2mE}{\hbar^2}\psi = 0$$

where E, m, and $\hbar$ are constants and $\psi = f(x, y, z)$. Separate the equation to an equation in x, an equation in y, and an equation in z by assuming that

$$\psi(x, y, z) = f(x)g(y)h(z)$$

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1. (a) $y = Ae^{-3x}$
   (b) $y = Ae^{3x}$
   (c) $y = c_1\, e^{-x} + c_2\, x\, e^{-x}$
   (d) $y = c_1\, e^{3x} + c_2\, x\, e^{3x}$
   (e) $y = A\, e^{i3x} + B\, e^{-i3x}$
   (f) $-\frac{1}{k_1 + k_2} \ln[k_1 a - (k_1 + k_2)x] = t + C$
   (g) $\phi = Ae^{-ar}$
   (h) $\ln(A) = -kt + C$
   (i) $\Phi = A\, e^{im\phi} + B\, e^{-im\phi}$
   (j) $y = A \sin \sqrt{\frac{k}{m}}\, t + B \cos \sqrt{\frac{k}{m}}\, t$
   (k) $\psi = A \sin \sqrt{\frac{8\pi^2 mE}{h^2}}\, x + B \cos \sqrt{\frac{8\pi^2 mE}{h^2}}\, x$
2. (a) Exact
   (b) Exact
   (c) Exact
   (d) Exact
   (e) Exact
   (f) Inexact
   (g) Exact
   (h) Exact
   (i) Inexact
   (j) Exact
   (k) Exact
5. $(\kappa^2 - c^2)a_\kappa = 0$
7. $f(x) = A \sin \frac{2\pi x}{\lambda} + B \cos \frac{2\pi x}{\lambda}$
8. $\lambda = \frac{2L}{n};\ n = 1, 2, 3, 4, \ldots$