

LABORATORIUM KIMIA FISIKA
Jurusan Kimia - FMIPA
Universitas Gadjah Mada (UGM)

MATEMATIKA KIMIA

Persamaan Differensial

(Sumber : Barrante, Applied Mathematics fo Physical Chemistry, Bab 6)

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1. Solve the following linear differential equations:

(a) $\frac{dy}{dx} + 3y = 0$

(b) $\frac{dy}{dx} - 3y = 0$

(c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

(d) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

(e) $\frac{d^2y}{dx^2} + 9y = 0$

(f) $\frac{dx}{dt} = k_1(a - x) - k_2x$; k_1 , k_2 , and a are constants.

(g) $\frac{d\phi}{dr} = -a\phi$; a is constant.

(h) $\frac{d(A)}{(A)} = -k dt$; k is constant.

(i) $\frac{1}{\Phi(\phi)} \frac{d^2\Phi}{d\phi^2} = -m^2$; m is constant.

(j) $m \frac{d^2y}{dt^2} = -ky$; m and k are constants.

(k) $\frac{d^2\psi}{dx^2} + \frac{8\pi^2mE}{h^2}\psi = 0$; E , m , and h are constants.



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2. Test the following differentials for exactness:

(a) $dF = 2xy^2 dx + 2yx^2 dy$

(b) $dF = 8x dx$

(c) $dF = 12x^2y dx + 4x^3 dy$

(d) $dF = 5 dx$

(e) $dF = \frac{1}{y} dx - \frac{x}{y^2} dy$

(f) $dF = xy dx + x^3 dy$

(g) $dP = \frac{nR}{V} dT - \frac{nRT}{V^2} dV$; n and R are constants.

(h) $dV = \pi r^2 dh + 2\pi rh dr$

(i) $dq = nC_v dT + \frac{nRT}{V} dV$; n , C_v , and R are constants.

(j) $d\rho = -\frac{PM}{RT^2} dT + \frac{M}{RT} dP$; M and R are constants.

(k) $dE = nC_v dT + \frac{n^2a}{V^2} dV$; n , C_v , and a are constants

3. Show that the differential $dq = nC_v dT + (nRT/V) dV$, where n , C_v , and R are constants, can be made exact by multiplying by an integrating factor $1/T$. The resulting differential dS is called the *differential entropy change*.



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3. Show that the differential $dq = nC_v dT + (nRT/V) dV$, where n , C_v , and R are constants, can be made exact by multiplying by an integrating factor $1/T$. The resulting differential dS is called the *differential entropy change*.
4. Show that if $\sin 3x$ and $\cos 3x$ are particular solutions to the differential equation

$$\frac{d^2y}{dx^2} + 9y = 0$$

then a linear combination of the two solutions also is a solution.

5. Bessel's equation is an important differential equation having the general form

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - c^2)y = 0$$

where c is a constant. Find the indicial equation for the series solution to this equation.

6. A one-dimensional harmonic oscillator is described classically by the equation

$$\frac{d^2y}{dt^2} + 4\pi^2\nu^2y = 0$$

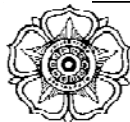
Show that a solution to this equation is $y = A \sin 2\pi\nu t$, where A , π , and ν are constants.

7. The differential equation describing the spacial behavior of a one-dimensional wave is

$$\frac{d^2f}{dx^2} + \frac{4\pi^2}{\lambda^2}f(x) = 0$$

where λ is the wavelength. Find the general solution to this equation.

8. *Boundary conditions* are special restrictions imposed on the solutions to differential equations. The boundary conditions for a plucked string bound at both ends between $x = 0$ and $x = L$, and described by the equation given in Problem 7, are that $f(x)$ goes to 0 at $x = 0$ and $x = L$. Show how these boundary conditions affect the solution to the equation in Problem 7.



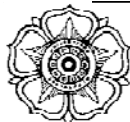
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9. Show that if we let $x = \cos \theta$, the solution to the associated Legendre's equation [Equation (6-47)] is $y = \cos \theta$, when $\ell = 1$ and $m = 0$, and is $y = \sin \theta$, when $\ell = 1$ and $m = 1$.
10. Show that if we let $x = \cos \theta$, the solution to the associated Legendre's equation [Equation (6-47)] is $y = \frac{1}{2}(3 \cos^2 \theta - 1)$, when $\ell = 2$ and $m = 0$.
11. The Schrödinger equation for a particle in a three-dimensional box is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2mE}{\hbar^2} \psi = 0$$

where E , m , and \hbar are constants and $\psi = f(x, y, z)$. Separate the equation to an equation in x , an equation in y , and an equation in z by assuming that

$$\psi(x, y, z) = f(x)g(y)h(z)$$



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1. (a) $y = Ae^{-3x}$

(b) $y = Ae^{3x}$

(c) $y = c_1 e^{-x} + c_2 x e^{-x}$

(d) $y = c_1 e^{3x} + c_2 x e^{3x}$

(e) $y = A e^{i3x} + B e^{-i3x}$

(f) $-\frac{1}{k_1 + k_2} \ln[k_1 a - (k_1 + k_2)x] = t + C$

2. (a) Exact

(b) Exact

(c) Exact

(d) Exact

(e) Exact

(f) Inexact

5. $(x^2 - c^2)a_x = 0$

7. $f(x) = A \sin \frac{2\pi x}{\lambda} + B \cos \frac{2\pi x}{\lambda}$

8. $\lambda = \frac{2L}{n}; n = 1, 2, 3, 4, \dots$

(g) $\phi = Ae^{-ar}$

(h) $\ln(A) = -kt + C$

(i) $\Phi = A e^{im\phi} + B e^{-im\phi}$

(j) $y = A \sin \sqrt{\frac{k}{m}}t + B \cos \sqrt{\frac{k}{m}}t$

(k) $\psi = A \sin \sqrt{\frac{8\pi^2 m E}{h^2}}x + B \cos \sqrt{\frac{8\pi^2 m E}{h^2}}x$

(g) Exact

(h) Exact

(i) Inexact

(j) Exact

(k) Exact

