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MATEMATIKA KIMIA
Deret Tak Terhingga

(Sumber : Barrante, Applied Mathematics fo Physical Chemistry, Bab 7)

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1. Using the comparison tests, determine whether each of the following series are convergent or divergent:

(a) $1 + 3 + 5 + 7 + 9 + \cdots$

(b) $\frac{3}{2} + \frac{3}{4} + \frac{3}{6} + \frac{3}{8} + \frac{3}{10} + \cdots$

(c) $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \frac{1}{4(5)} + \cdots$

(d) $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \cdots$

(e) $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots$

(f) $\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \cdots$

(g) $1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \cdots$

(h) $\frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \frac{1}{108} + \frac{1}{324} +$

(i) $\frac{1}{2} + \frac{2!}{2^2} + \frac{3!}{2^3} + \frac{4!}{2^4} + \cdots$

(j) $1 + \frac{1}{3} + \frac{1}{4^2} + \frac{1}{5^3} + \frac{1}{6^4} + \cdots$

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2. Using the ratio test, determine whether the following series are convergent or divergent:

(a) $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\frac{1}{2^5}+\cdots$

(b) $3+\frac{3^2}{2}+\frac{3^3}{3}+\frac{3^4}{4}+\frac{3^5}{5}+\cdots$

(c) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\cdots$

(d) $2+\frac{2^2}{2^2}+\frac{2^3}{3^2}+\frac{2^4}{4^2}+\frac{2^5}{5^2}+\cdots$

(e) $\frac{1}{2}+\frac{2^2}{2^2}+\frac{3^2}{2^3}+\frac{4^2}{2^4}+\frac{5^2}{2^5}+\cdots$

(f) $3+\frac{3^2}{2!}+\frac{3^3}{3!}+\frac{3^4}{4!}+\frac{3^5}{5!}+\cdots$

(g) $1+\frac{1!}{2}+\frac{2!}{3}+\frac{3!}{4}+\frac{4!}{5}+\cdots$

(h) $\frac{1}{3!}+\frac{1}{6!}+\frac{1}{9!}+\frac{1}{12!}+\frac{1}{15!}+\cdots$

(i) $1+\frac{1}{2}+\frac{2!}{2^2}+\frac{3!}{2^3}+\frac{4!}{2^4}+\cdots$

(j) $\frac{1}{3}+\frac{2^2}{4}+\frac{3^2}{5}+\frac{4^2}{6}+\frac{5^2}{7}+\cdots$

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3. Determine the interval of convergence for the following power series:

(a) $1 + x + x^2 + x^3 + \cdots$

(b) $1 - 2x + 3x^2 - 4x^3 + - \cdots$

(c) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

(d) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + - \cdots$

(e) $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + - \cdots$

(f) $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + - \cdots$

(g) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - \cdots$

(h) $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + - \cdots$

(i) $1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \frac{x^4}{16} + \cdots$

(j) $1 + (x+2) + (x+2)^2 + (x+2)^3 + \cdots$

4. Expand the following functions in a Maclaurin series:

(a) $\frac{1}{1+x}$

(b) $\frac{1}{(1-x)^2}$

(c) $(1+x)^{1/2}$

(d) $\ln(1-x)$

(e) e^{-x^2}

(f) a^x

(g) $\cos x$

(h) $(x+1)^3$

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5. Show that, for small values of X_B, $\ln(1-X_B) \cong -X_B$.
6. Show that, for small values of θ, $\sin\theta \cong \theta$.
7. Show, by expanding $\sin x$ in powers of $(x-a)$, that the series converges most rapidly as x approaches a.
8. Evaluate the integral $\int_0^4 e^{-x^2}dx$ by expanding the function in a Maclaurin series (first 8 terms).
9. Show that the solutions to the particle in the one-dimensional box, $\psi_n = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, are orthogonal and normalized.
10. Find the Fourier transform of the function

$$f(x) = \begin{cases} 0; & x < -\pi \\ x; & -\pi < x < \pi \\ 0; & x > \pi \end{cases}$$

11. Find the Fourier transform of the step function

$$f(x) = \begin{cases} 0; & x < -L \\ \frac{\sqrt{2\pi}}{2L}; & -L < x < L \\ 0; & x > L \end{cases}$$

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12. Of the Fourier transforms that occur throughout mathematics, chemistry, and physics, perhaps the most striking are those that occur in the areas of diffraction. For example, we can use a Fourier transform to reorganize the information found in an X-ray diffraction pattern and retransform it back into an "image" of a crystal. Consider the following one-dimensional crystal structure problem:

$$F_0 = +52.0 \qquad F_3 = +25.8$$
$$F_1 = -20.0 \qquad F_4 = -8.9$$
$$F_2 = -14.5 \qquad F_5 = -7.2$$

For a centrosymmetric wave (that is, a wave that is symmetric about the region of space in which it exists), the Fourier series is

$$f(x) = F_0 + \sum_n F_n \cos 2\pi n x \qquad (n = 1 \text{ to } \infty)$$

where F_i represents the Fourier coefficients given above. Plot $f(x)$ from $x = 0$ to $x = 1$ in steps of 0.05 and show that the first six terms approximate a one-dimensional unit cell containing two atoms, one at $x = 1/3$ and the other at $x = 2/3$.

13. The step function

$$f(x) = \begin{cases} -1; & -\pi \le x < 0 \\ +1; & 0 < x \le \pi \end{cases}$$

can be described by the Fourier series $f(x) = \sum_n a_n \sin nx$, where

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

Plot the actual function from $-\pi$ to $+\pi$ and compare it to a plot of the Fourier series containing the first five nonzero terms.

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1. (a) divergent
   (b) divergent
   (c) convergent
   (d) convergent
   (e) convergent
   (f) divergent
   (g) divergent
   (h) convergent
   (i) divergent
   (j) convergent

2. (a) convergent
   (b) divergent
   (c) test fails
   (d) divergent
   (e) convergent
   (f) convergent
   (g) divergent
   (h) convergent
   (i) divergent
   (j) test fails

3. (a) $-1 < x < 1$
   (b) $-1 < x < 1$
   (c) all values of x
   (d) $-1 < x \leq 1$
   (e) all values of x
   (f) $-1 \leq x \leq 1$
   (g) all values of x
   (h) $0 < x \leq 2$
   (i) $-2 < x < 2$
   (j) $-3 < x < -1$

4. (a) $1 - x + x^2 - x^3 + - \cdots$
   (b) $1 + 2x + 3x^2 + 4x^3 + \cdots$
   (c) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - + \cdots$
   (d) $-x - x^2/2 - x^3/3 - x^4/4 - \cdots$
   (e) $1 - x^2 + x^4/2! - x^6/3! + - \cdots$
   (f) $1 + x \ln a + (x \ln a)^2/2! + (x \ln a)^3/3! + \cdots$
   (g) $1 - x^2/2! + x^4/4! - x^6/6! + - \cdots$
   (h) $1 + 3x + 3x^2 + x^3$

10. $g(k) = \frac{2i}{\sqrt{2\pi}k^2}(k\pi \cos k\pi - \sin k\pi)$

11. $g(k) = \frac{\sin kL}{kL}$