LABORATORIUM KIMIA FISIKA
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MATEMATIKA KIMIA
Matrik dan Determinan

(Sumber : Barrante, Applied Mathematics fo Physical Chemistry, Bab 9)

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1. Evaluate the following determinants:

(a) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ **(b)** $\begin{vmatrix} 6 & 1 \\ -1 & -1 \end{vmatrix}$ **(c)** $\begin{vmatrix} 4 & -3 \\ 0 & -1 \end{vmatrix}$

(d) $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ **(e)** $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ **(f)** $\begin{vmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{vmatrix}$

(g) $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \\ -1 & 4 & 2 \end{vmatrix}$ **(h)** $\begin{vmatrix} 4 & 2 & -1 \\ -1 & 6 & 3 \\ -1 & 5 & -1 \end{vmatrix}$ **(i)** $\begin{vmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{vmatrix}$

(j) $\begin{vmatrix} 4 & 3 & 1 & -1 \\ 6 & 1 & 0 & -3 \\ 1 & 5 & 2 & -2 \\ 8 & 6 & -5 & 0 \end{vmatrix}$ **(k)** $\begin{vmatrix} x & b & 0 & 0 \\ b & x & b & 0 \\ 0 & b & x & b \\ 0 & 0 & b & x \end{vmatrix}$

2. Solve the following determinants for x:

(a) $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = 0$ **(b)** $\begin{vmatrix} x & -2 \\ 1 & x \end{vmatrix} = 6$ **(c)** $\begin{vmatrix} 2x & 4 \\ 2 & x \end{vmatrix} = 2$

(d) $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = 2$ **(e)** $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 0 & 0 \\ 1 & 0 & x & 0 \\ 1 & 0 & 0 & x \end{vmatrix} = 0$

3. Add the matrices:

$$\begin{pmatrix} 1 & 1 & 4 & 3 \\ -1 & 0 & 1 & 2 \\ -1 & 2 & 4 & -3 \\ 5 & 6 & 3 & 5 \end{pmatrix} + \begin{pmatrix} 4 & 0 & -4 & 3 \\ 6 & 3 & -7 & 5 \\ -1 & 1 & -1 & 0 \\ -5 & 2 & 6 & 7 \end{pmatrix}$$

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4. Perform the following matrix multiplication:

(a) $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 6 & -3 \\ -3 & 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 0 & 3 \\ 4 & -1 & -1 \\ 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & 6 \\ 3 & 4 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 4 & 1 \\ 0 & 1 & 2 \\ 2 & 4 & -3 \end{pmatrix}\begin{pmatrix} 0 & -4 & 3 \\ 6 & 3 & -7 \\ 2 & 6 & 7 \end{pmatrix}$ **(e)** $\begin{pmatrix} 1 & 8 & 4 \\ -2 & 3 & 0 \\ 5 & -1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

5. Given the two matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 4 \\ 2 & -6 & 10 \\ 4 & -1 & -1 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 6 & 1 & 0 \\ 4 & 2 & -1 \\ 8 & -4 & 3 \end{pmatrix}$$

show that $\mathbf{A\,B} \neq \mathbf{B\,A}$.

6. Solve the following sets of equations using Cramer's rule:

(a)
$$\begin{aligned} x + y &= 3 \\ 4x - 3y &= 5 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y + 3z &= -5 \\ -x - 3y + z &= -14 \\ 2x + y + z &= 1 \end{aligned}$$

(c)
$$\begin{aligned} x + 2y - z + t &= 2 \\ x - 2y + z - 3t &= 6 \\ 2x + y + 2z + t &= -4 \\ 3x + 3y + z - 2t &= 10 \end{aligned}$$

(d)
$$\begin{aligned} x\sin\theta + y\cos\theta &= x' \\ -x\cos\theta + y\sin\theta &= y' \end{aligned}$$

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7. Show that only a trivial solution is possible for the set of equations:

$$x + y = 0$$

$$x - y = 0$$

8. Show that the matrix

$$\mathbf{E} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

will transform the vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ into itself.

9. Show that the matrix

$$\mathbf{C}_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

will transform the vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ into $\begin{pmatrix} -x \\ -y \\ z \end{pmatrix}$.

10. Prove that the inverse of the matrix

$$\mathbf{C}_3 = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{is} \quad \mathbf{C}_3^{-1} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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11. Put the following matrix in diagonal form:

$$A = \begin{pmatrix} 1 & \sqrt{6} \\ \sqrt{6} & 2 \end{pmatrix}$$

12. Show that the eigenvectors

$$\mathbf{C} = \begin{pmatrix} \frac{\sqrt{6}}{\sqrt{15}} & \frac{\sqrt{6}}{\sqrt{10}} \\ \frac{3}{\sqrt{15}} & \frac{-2}{\sqrt{10}} \end{pmatrix}$$

will diagonalize matrix **A** in Problem 11. (*Hint*: show by matrix multiplication that $\mathbf{C}^{-1}\mathbf{AC} = \mathbf{\Lambda}$, where $\mathbf{\Lambda}$ is the diagonal form of **A**.

13. Show that **c** and λ**c**, where λ is a scalar, are parallel vectors in space.

14. Solve the following set of secular equations for E in terms of α and β. Determine the relationship between the coefficients c_1, c_2, and c_3 for each value of E, and using the fact that $\sum c_i^2 = 1$ (that is, that the eigenvectors must be normalized), find the values of c_1, c_2, and c_3 for each value of E.

$$\begin{aligned} (\alpha - E)c_1 + \beta c_2 + \beta c_3 &= 0 \\ \beta c_1 + (\alpha - E)c_2 + \beta c_3 &= 0 \\ \beta c_1 + \beta c_2 + (\alpha - E)c_3 &= 0 \end{aligned}$$

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1. (a) -2 (b) -5 (c) -4 (d) 1 (e) $x^2 - 1$ (f) 1 (g) 18 (h) -93 (i) $x^3 - 2x$ (j) $+352$ (k) $x^4 - 3x^2b^2 + b^4$

2. (a) $x = \pm 1$ (b) $x = \pm 2$ (c) $x = \pm\sqrt{5}$ (d) $x = 0, \pm\sqrt{3}$ (e) $x = 0, 0, \pm\sqrt{3}$

3. $\begin{pmatrix} 5 & 1 & 0 & 6 \\ 5 & 3 & -6 & 7 \\ -2 & 3 & 3 & -3 \\ 0 & 8 & 9 & 12 \end{pmatrix}$

4. (a) $\begin{pmatrix} -6 & 1 \\ 12 & -7 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 12 & 15 & 18 \\ 3 & -1 & -7 \\ 12 & 23 & 38 \end{pmatrix}$

(d) $\begin{pmatrix} 26 & 14 & -18 \\ 10 & 15 & 7 \\ 18 & -14 & -43 \end{pmatrix}$ (e) $\begin{pmatrix} x + 8y + 4z \\ -2x + 3y \\ 5x - y - z \end{pmatrix}$

5. $\begin{pmatrix} 42 & -13 & 11 \\ 68 & -50 & 36 \\ 12 & 6 & -2 \end{pmatrix}$ vs $\begin{pmatrix} 8 & 0 & 34 \\ 4 & -7 & 37 \\ 12 & 29 & -11 \end{pmatrix}$

6. (a) $x = 2, y = 1$

(b) $x = 1, y = 3, z = -4$

(c) $x = 1, y = 1, z = -2, t = -3$

(d) $x = x' \sin\theta - y' \cos\theta$

$y = y' \sin\theta + x' \cos\theta$

11. $\mathbf{A} = \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}$

14. $E_1 = \alpha + 2\beta$; $E_2 = E_3 = \alpha - \beta$

For E_1: $c_1 = c_2 = c_3 = \dfrac{1}{\sqrt{3}}$

For E_2: $c_1 = \dfrac{1}{\sqrt{2}}, c_2 = -\dfrac{1}{\sqrt{2}}, c_3 = 0$

For E_3: $c_1 = \dfrac{1}{\sqrt{6}} = c_2, c_3 = -\dfrac{2}{\sqrt{6}}$